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Essential work of fracture: Simulating the toughness response of deeply double-edge notched specimens

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Abstract

The essential work of fracture technique although straightforward, is time and material demanding. A non-linear logarithmic function was proposed for the simulation of the force-elongation behaviour of the deeply double edge-notched specimens used in the experimental procedure for the essential work of fracture technique. A variety of blends of polyolefin based matrices with different rubber types, and also kaolin particles were used to validate the proposed function. The empirical function used for the simulation gave satisfactory results over a range of crack sizes.

Introduction

Before presenting the main part of this work, which is the attempt to simulate the load-displacement response of the deeply double edge-notched (DDEN-T) specimens it would be useful to give here a brief introduction into the essential work of fracture theory.

Essential Work of Fracture Theory

According to the EWF theory (1-3), a distinction is being made between a process zone or process plane where the actual crack runs, and a plastic zone, which surrounds the process zone. Consequently, the total work required to fracture a pre-cracked specimen can also be divided in two parts associated with each of the two zones mentioned above. It can be written therefore:

$$W_f = W_e + W_p \quad [1]$$

where W_f is the total fracture work, W_e the work spent for the crack advance in the process plane (generation of new surfaces) and W_p the energy consumed in the plastic zone. Thus, W_e is related to a 2-D plane and is therefore a function of area (lt), whereas W_p is dissipated in a 3-D plastic zone and can be thus considered a function of volume (l^2t), where: t=specimen thickness, l=ligament. Equation 1 can be also expressed by the specific terms:

$$w_f = w_e + ßw_p l \quad [2]$$

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where: $w_f = W_f/lt$, $w_p, = W_p/l^2t$, and ß is a geometry factor associated with the shape of the plastic zone.
According to Equation 2, the work of fracture is a linear function of the ligament size. w_e can be determined from the interception of the linear regression line, fitted to the w_f vs l graphs, with the y-axis. It should be mentioned here that W_f can be determined by calculating the integral of force over displacement from the tensile tests performed on deeply double edge notched tensile (DDEN-T, cf. Figure 1) specimens of various ligament sizes. An important prerequisite of the plane stress EWF approach is that the crack propagates only after the ligament has been fully yielded.

Simulation necessity

Up to this point, and according to the ESIS testing protocols (4), at least 20 specimens are needed to obtain valid EWF data results. In combination with the fact that DDEN-T specimens can be quite large, it is easy to conclude that the EWF testing method is very material demanding. Moreover, in most cases the crosshead speed during tests is slower than 5mm/min, which implies a long testing period for all specimens. An alternative way is needed to validate and/or obtain EWF results without undertaking the whole arduous experimental procedure used up to now.

As stated in previous communications (5-8), one of the most important EWF method prerequisites is the self-similarity of the force-elongation (F-X) plots obtained during fracture of the DDEN-T specimens. It has been observed that the EWF method works perfectly when these curves have similar shape characteristics. On the contrary, plots without similarities lead to large experimental scatter as seen in previous work (9-10).

Self-similarity means that the F-X plots for a material depend on the actual ligament length. As we can see in Figure 2 for the example of a rubber toughened polypropylene, the plots are similar to each other and almost analogue to ligament size. So the quantity:

$$W_f = \oint F dx \quad [3]$$

which represents the work of fracture or total energy required to tear a specimen apart, should also be a linear function of ligament size.

The latter conclusion implies that a function describing the F-X or F(x) plots is enough to obtain the work of fracture of a DDEN-T specimen for any ligament size, within the EWF requirement for plain stress fracture: l>3t. It is therefore of paramount siginificance to undertake effort on simulation. This would allow us to obtain valid specific essential work of fracture data within short times and by using a minimum number of specimens.

Materials and specimen preparation

A variety of materials were used for the experimental procedures. Blends of two different kinds of thermoplastic elastomers with metallocene catalyzed isotactic polypropylene (m-iPP) were chosen for the experimental study/empirical simulation. The elastomers were ethylene-co-butylene copolymers of a varying 1-butylene content (EBR). Type EBR-90 had an almost 90wt% in 1-butylene in comparison to the EBR-48 type which contained only 48wt%. Blend composition is as follows: m-iPP/EBR90; 85/15 wt% and m-iPP/EBR-48; 85/15 wt% also. Both blends were available in compression molded disk-shaped plates of 130 mm diameter and 1 mm thickness (11).

Injection molded plaques of a poly(butylene terephthalate) (PBT) blend with a functionalized ethylene-acrylate elastomer (EAF) were provided by BASF AG (Ludwigshafen, Germany) in form of injection molded square plates of 180^*180^*4 mm^3. Blend weight ratio was set at 80/20% in PBT/EAF particles respectively. A blend of high density polyethylene (HDPE) with kaolin ($Al_2O_3 \cdot SiO_2 \cdot 2H_2O$) particles at a mixture ratio of 70/30 wt%, respectively, was also included in this study. This material was available in the form of injection molded plaques of 3 mm thickness.

DDEN-T specimens (dimensions: 40 mm x 80 mm, cf. Figure 1) for the EWF tests were cut from all above mentioned blend plaques by a rotating disk table-saw. All specimens were precracked using a band saw and a fine notch was introduced afterwards by tapping with a fresh razor blade. All tests were performed at a crosshead speed of 1 mm./min. The primary EWF data are available from previous works (11-12) where the toughness response of the blends tested was discussed in detail.

Results and discussion

Simulation process

Figure 2 displays the force-displacement response of a series of DDEN-T specimens of m-iPP/15wt% EBR48. There are some interesting observations which can be made on this diagram. As we can see, the deformation of the specimen corresponding to the load maximum appears to be a constant value, except for large ligaments. In other words, the deformation at yield point, X_y, changed at a very small scale for many of the specimens used. X_y was experimentally determined by using 3-5 DDEN-T specimens.

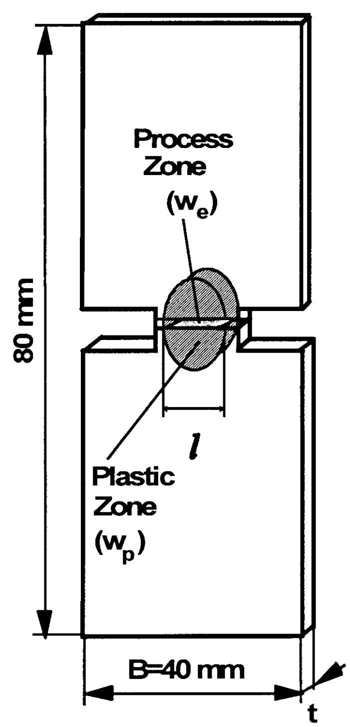

Figure 1. A DDEN-T specimen

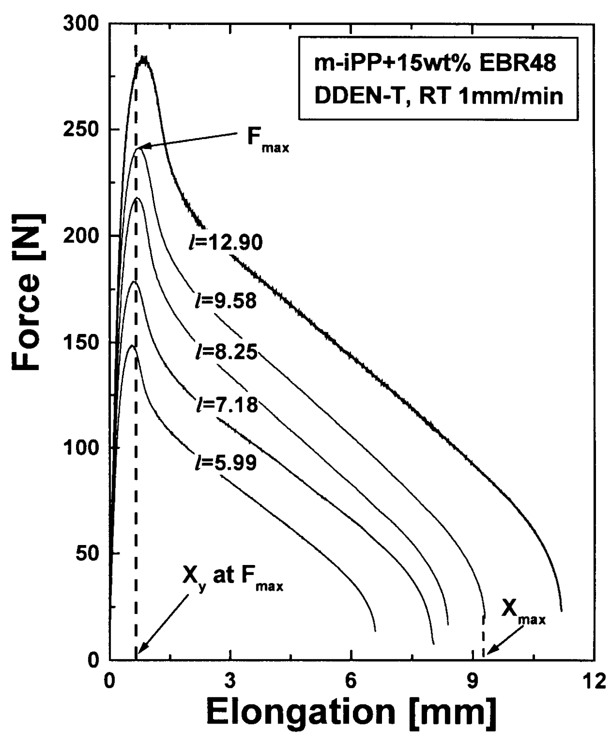

Figure 2. F-X curves for the m-iPP+15wt% EPR48 DDEN-T specimens

It is well known from Hill's plasticity theory (13), that the net section stress (σ_n) at the load maximum (yield point) for DDEN-T specimens should be: $\sigma_n \leq 1.15\sigma_y$ (4,13). Taking into account that for each specimen the net section (notched section, A) is a known quantity, it can be easily concluded that the maximum load, F_{max}, for each specimen can also be predicted exactly and will be: $F_{max} = \sigma_n A$ [5], where A is the ligament cross-section ($A=lt$). The empirical function proposed for the simulation of the force-displacement behaviour of the DDEN-T specimens is a normalized logarithmic one, of the type:

$$y(x) = D\exp[-\ln^2 \frac{(x/x_c)}{2w^2}] \qquad [6]$$

This kind of function can describe single peak phenomena (i.e. onset of yield and fracture) quite well. It is a distribution function with a variety of applications in medicine, economics, environmental and social sciences (14-15). This function requires the determination of 3 constants: D, x_c and w respectively. It can be easily shown (cf. Figure 3) that when Equation 6 is fitted to the experimental F(x) graphs of DDEN-T specimens, x_c can be identified as the X_y quantity mentioned above, and the constant D is none other than F_{max} (see Fig.2). Equations 4,5, and 6 will now yield:

$$F(x) = 1.15\sigma_y lt \cdot \exp[-\ln^2 \frac{(x/X_y)}{2w^2}] \qquad [7]$$

In Figure 3 some applications of Equation 7, for the materials tested are shown. It is easy to conclude from the presented graphs that the proposed function works quite well especially in the area about the maximum load. However, some tail effects are visible for all specimens tested. It can be seen in Figure 3 that the ligament in the cases of the PBT and HDPE blends does not yield fully. Instead, ligament yielding takes place simultaneously with crack advance. Note, that this behaviour is common among polymers and polymer blends studied according to the EWF concept until now (16-20). However, since the work of fracture as given by Eq. 3 is an integral of the proposed Function 7, it is also expected that small deviations in the actual F(x) curves will be smoothened by the integration.

As already mentioned, a very important prerequisite of the EWF approach is based on the self similarity of the F-X curves. This however, implies that size of these curves is related to specimen ligament. Thus, it would be reasonable to suppose that the maximum elongation X_{max} of each DDEN-T specimen is a linear function of ligament, l. Moreover, the ratio of $\alpha = X_{max}/l$ should be the same for all specimens, unless a deviation from the curve similarity criterion would be stated.

Table 1. Constants determined from the DDEN-T specimens and yield strength

	$x_c = X_y$ [mm]	w [-]	$\alpha = X_{max}/l$ [-]	σ_y [MPa]
HDPE/kaolin	0.39	1.08	0.18	30.1
PP/EBR48	0.93	1.42	1.08	19.7
PP/EBR90	0.70	1.53	1.06	19.4
PBT/EAF	0.54	1.34	0.46	32.0

During the application of Eq.6 for the curve fits of Figure 3 it was observed that parameter *w* remained almost constant for each specimen series (Table 1).
Based on the three specimens of different ligaments for each blend, the average values obtained for the X_y, and w constants of Eq. 7, and also the $\alpha = X_{max}/l$ ratio along with the yield strength, are given in Table 1.

It is important to notice that the elongation X_y is not always an almost contant value like in the case of the materials tested. Indeed, it was pointed out in other studies (21,22) that X_y is also a linear function of the specimen ligament. In this case, Equation 7 is still valid, provided that the term X_y is substituted by the term: $\alpha \cdot l$, where α is none other than the same analogy ratio as given in Table 1 for X_{max}.

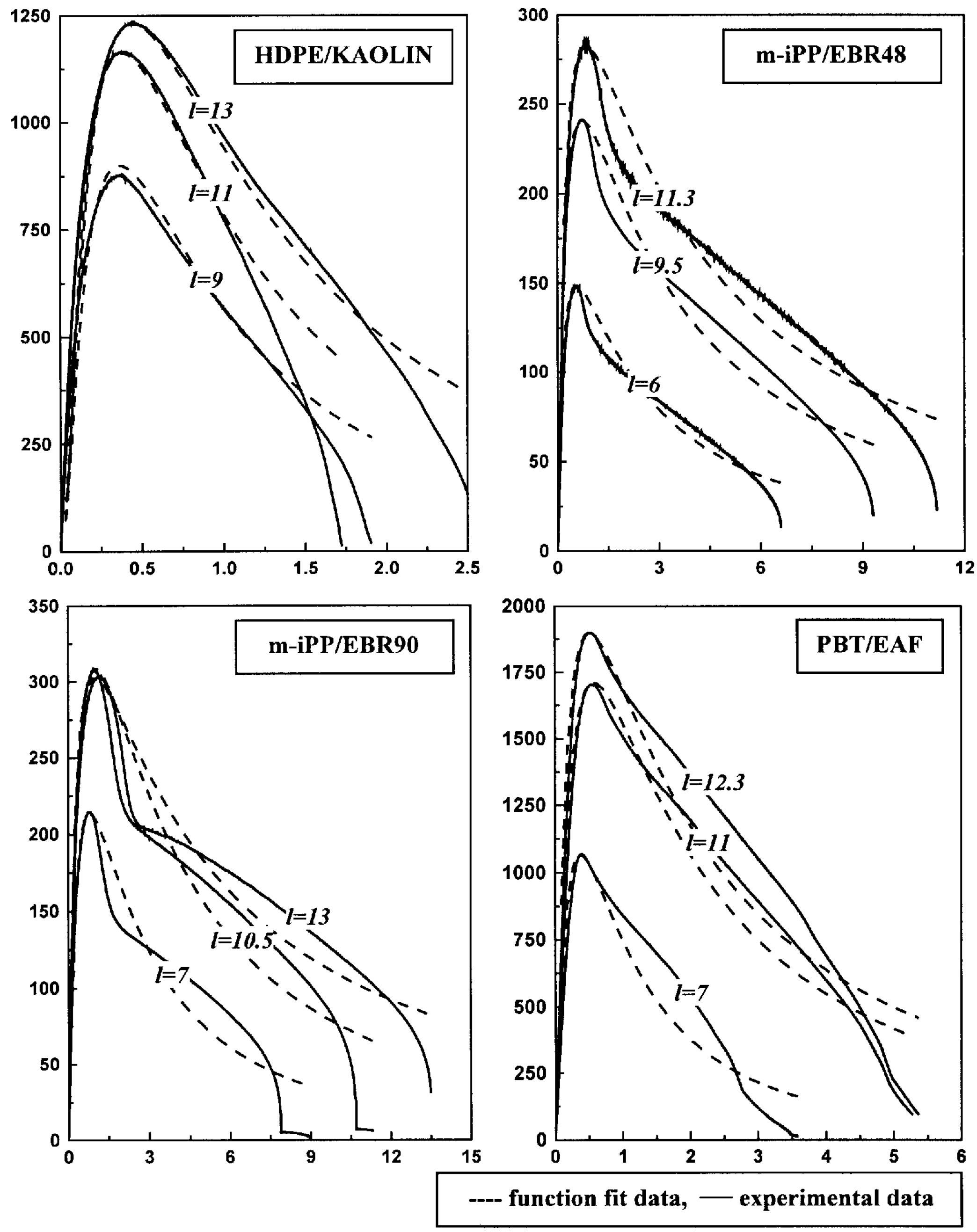

Figure 3. Application of the empirical fit function for the materials tested. l [mm], Y-axis: force [N], X-axis: displacement [mm]

By substituting X_y by $\alpha \cdot l$ in Equation 7 we get:

$$F(x) = 1.15\sigma_y lt \cdot \exp[-\ln^2 \frac{(x/\alpha \cdot l)}{2w^2}] \quad [8]$$

Having determined the constants $x_c(=X_y)$, w, and the X_{max}-to-l ratio, α, it is very easy to apply Equation 7 over a series of different specimen ligaments for each of the polymer blends concerned in this study. The procedure was comprised of the following steps: for each specimen ligament l, F_{max} and X_{max} were calculated using Equation 5 and Table 1 respectively. From the same Table 1, $x_c(=X_y)$, could be read, too. In this way all three needed constants could be determined and the F(x) curve was plotted for the corresponding ligament.

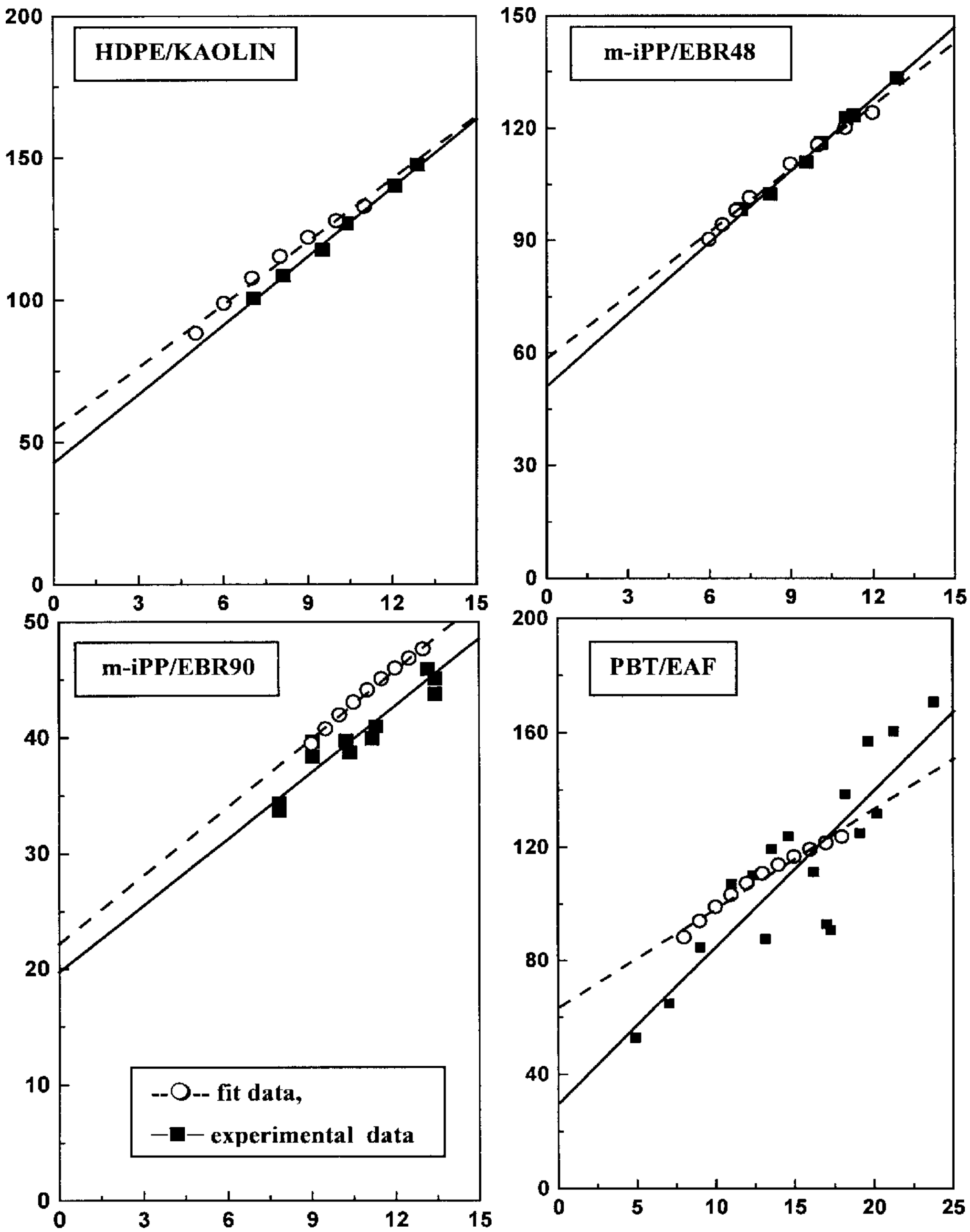

Figure 4. The w_f vs *l* plots and related linear regression lines for the materials studied. Y-axis: w_f [kJ/m^2], X-axis: *l* [mm]

Table 2. Essential Work of Fracture Parameters

	w_e [kJ/m^2]		βw_p [MJ/m^3]	
	Experimental	**Simulation**	**Experimental**	**Simulation**
HDPE/kaolin	19.72	22.18	1.92	1.96
PP/EBR48	50.99	58.42	6.39	5.61
PP/EBR90	42.79	54.35	8.06	7.56
PBT/EAF	29.78	63.64	5.48	3.47

The assigned work of fracture could be then calculated by integrating between 0 and X_{max}, force over displacement, as defined in Equation 3:

W_f can be then normalized by the ligament crossection: A=*l*t and thus the specific work of fracture, w_f, is obtained. It is evident, that by repeating the above steps for a series of specimen ligaments, one obtains the w_f-*l* pairs needed for the specific essential work of fracture plots. Consequently, by extrapolating the linear regression line for these graphs back to zero, both w_e and ßw_p can be determined. In Figure 4 the related experimental and simulated specific work of fracture vs ligament plots for the materials studied can be seen. The EWF data both essential and non-essential as determined by the experimental and simulation data are given in Table 2.

It can be seen from Table 2 and related Figure 4 that the method is more accurate for ductile systems under plane stress conditions. In this way, both the specific essential and non-essential works of fracture can be determined with high accuracy. However, there are cases like the PBT/EAF blends where the experimental results and the ones obtained by applying Equation 7 differ substantially. Probably more specimens were needed for the evaluation of the constants of Eq. 7 in this case, or the function is not applicable at all due to the relatively large experimental scatter observed for this material. This is probably due to the high thickness (4 mm) and the mixed mode (plane stress/plane strain) of fracture reported for PBT blends in a previous communication (23).

Of course, the authors understand that the normalized logarithmical simulation proposed cannot describe the experimental F(x) curves point by point for all the elongation undergone by a DDEN-T specimen. This is also evident in our Figure 3 where we show the difference between the simulated and experimental F(x) curves for all materials tested. Please note here however that the work of fracture W_f depends on the integral of the F(x) as Equation 3 denotes. However, the ability of the normalized logarithmical function proposed to provide in means of integration very good results, as far as the W_f is concerned, is also evident in Figure 4. Actually, the procedure of extracting the toughness parameters or the integration results was not checked by the referee either, who restrained only in comparing the shapes of the experimental and simulated F(x) curves.

It is obvious from Figure 4 and Table 2 that the normalized logarithmic function can approach quite well the essential work of fracture parameters, even if the shape of the simulated F(x) curve is not 100% the same with the actual experimental as Figure 3 shows. Moreover, our main goal in this work was not to simulate the exact performance in means of force-displacent of the DDEN-T specimens but the toughness response in terms of the essential work of fracture method and we have shown (cf. Figure 4 and Table 2) that this was feasible.

It is also true, that many times during the experimental results, DDEN-T specimens do not follow Hill's rule $\sigma_n \leq 1.15\sigma_y$. Instead, the yield stress measured for them lays around the $1.15\sigma_y$ limit (4). Hill's criterion however, provides us with a prediction tool for obtaining the $D=F(X_y)$ parameter over a series of ligaments.

Conclusions

An empirical lognormal function is proposed for the simulation of the deformation behaviour of the DDEN-T specimens. The simulation accuracy was quite high over a range of crack sizes. It was found that the function works quite well for plane stress fracture conditions. This function offers a useful alternative to the traditional EWF experimental technique, when the curve similarity criterion is valid. This allows the exact calculation of the lognormal fit parameters. Consequently, the determination of the essential work of fracture parameters is possible at a given specimen ligament range. The fit method proposed delivers fair results in the case of mixed mode fracture of the DDEN-T specimens, where large experimental scatter is observed.

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